Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 60 PTS**

[a] $\int_{-1}^{1} (y^2 \tan y - 3\sqrt{1-y^2})\, dy = \frac{-3\pi}{2}$ 2

15

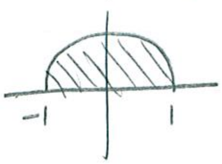
$= \int_{-1}^{1} y^2 \tan y \, dy - 3\int_{-1}^{1} \sqrt{1-y^2}\, dy$ 3

$y^2 \tan y$ IS CONTINUOUS ON $[-1,1]$ 2

$(-y)^2 \tan(-y) = -y^2 \tan y$, 2

SO INTEGRAND IS ODD, $\int = 0$ 2 2

$\int_{-1}^{1} \sqrt{1-y^2}\, dy = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ 2



[b] $\int_{-2}^{2} (10x-22)\sqrt{5-2x}\, dx$ SEE ALTERNATE SOLUTION ON OTHER KEYS

20

$u = 5-2x$ 3 $x = 2 \rightarrow u = 1$, $x = -2 \rightarrow u = 9$ $x = \frac{5-u}{2}$ 3

$\frac{du}{dx} = -2 \rightarrow dx = -\frac{1}{2}du$

$(10x-22)\sqrt{5-2x}\, dx$

$= -\frac{1}{2}(10x-22)\sqrt{5-2x}\, du$ 3

$= -\frac{1}{2}(10 \cdot \frac{5-u}{2} - 22)\sqrt{u}\, du$

$= -\frac{1}{2}(3-5u)u^{\frac{1}{2}}\, du$

$= (-\frac{3}{2}u^{\frac{1}{2}} + \frac{5}{2}u^{\frac{3}{2}})\, du$ 3

3 $\int_{9}^{1} (-\frac{3}{2}u^{\frac{1}{2}} + \frac{5}{2}u^{\frac{3}{2}})\, du$

$= (-u^{\frac{3}{2}} + u^{\frac{5}{2}})\Big|_{9}^{1}$ 3

$= (-1^{\frac{3}{2}} + 1^{\frac{5}{2}}) - (-9^{\frac{3}{2}} + 9^{\frac{5}{2}}) = -(-27+243)$

$= -216$ 2

[c] $\int_{-\pi}^{\pi} \theta^2 \sec\theta \, d\theta$

10

$\theta^2 \sec\theta$ IS DISCONTINUOUS 4

@ $\theta = \pm\frac{\pi}{2}$ 3

FTC DOES NOT APPLY 3

[d] $\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{3 - 6\cos t}{\sin^2 t}\, dt$

15

$= \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} (3\csc^2 t - 6\csc t \cot t)\, dt$ 5

$= (-3\cot t + 6\csc t)\Big|_{\frac{\pi}{6}}^{\frac{2\pi}{3}}$ 5

$= (-3 \cdot \frac{-\sqrt{3}}{3} + 6 \cdot \frac{2\sqrt{3}}{3})$

$- (-3 \cdot \sqrt{3} + 6 \cdot 2)$

$= (\sqrt{3} + 4\sqrt{3}) - (-3\sqrt{3} + 12)$

$= 8\sqrt{3} - 12$ 5

Using proper English and mathematical notation, state all parts of the Fundamental Theorem of Calculus. (The Net Change Theorem is one of the parts.) **SCORE: _____ / 15 PTS**

SEE SOLUTION OF QUIZ 3

Find $\frac{d}{dx}(x^2 \cosh^{-1} 4x + \operatorname{sech} \sqrt[3]{x})$. **SCORE: _____ / 15 PTS**

You may use any hyperbolic identities or the derivatives of any hyperbolic functions without proving them.

$$2x\cosh^{-1}4x + x^2 \frac{1}{\sqrt{(4x)^2-1}}(4) - \operatorname{sech}\sqrt[3]{x}\tanh\sqrt[3]{x} \cdot \left(\frac{1}{3}x^{-\frac{2}{3}}\right)$$

$$= 2x\cosh^{-1}4x + \frac{4x^2}{\sqrt{16x^2-1}} - \frac{\operatorname{sech}\sqrt[3]{x}\tanh\sqrt[3]{x}}{3\sqrt[3]{x^2}}$$

3 EACH

If f is continuous and $\int_{-1}^{5} f(t)\,dt = 6$, find $\int_{-1}^{2} (5+4f(3-2t))\,dt$. **SCORE: _____ / 15 PTS**

3 $u = 3-2t$; $t=2 \rightarrow u=-1$; $t=-1 \rightarrow u=5$

$\frac{du}{dt} = -2$

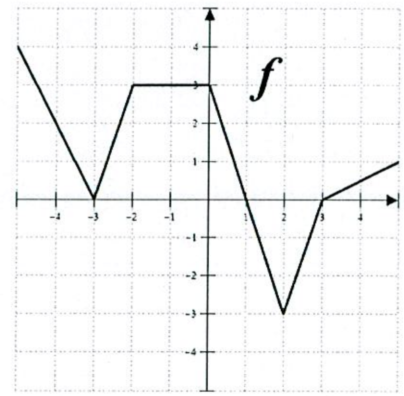
$dt = -\frac{1}{2}du$

2 $\left[\int_5^{-1} -\frac{1}{2}(5+4f(u))\,du\right.$ 2 $= \int_5^{-1} -\frac{5}{2}\,du - 2\int_5^{-1} f(u)\,du$ 2

2 $= -\frac{5}{2}(-1-5) + 2\int_{-1}^{5} f(u)\,du$ 2 $= 15 + 2(6)$

$= 27$ 2

Let $g(x) = \int_{-4}^{x} f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 30 PTS**



[a] Find $g(2)$.

9

$$\underbrace{\int_{-4}^{2} f(t)\,dt = \int_{-4}^{1} f(t)\,dt + \int_{1}^{2} f(t)\,dt}_{}\ 3$$

$$= \tfrac{1}{2}(1)(2) + \tfrac{1}{2}(2+4)(3) - \tfrac{1}{2}(1)(3)$$

$$= \underbrace{1}_{1} + \underbrace{9}_{2} - \underbrace{\tfrac{3}{2}}_{2} = \underbrace{\tfrac{17}{2}}_{1}$$

[b] Find $g'(2)$. **Explain your answer very briefly.**

$$g'(2) = \underbrace{f(2) = -3}_{3}$$

[c] Find all local minima of g. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) \text{ CHANGES FROM NEGATIVE TO POSITIVE}}$ 4

@ $\underbrace{x = 3}$ 4

[d] Find all inflection points of g. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) \text{ CHANGES FROM DECREASING TO INCREASING}}$ 4

@ $\underbrace{x = -3, 2}_{6}$

Prove that $\pi^2 \le \int_{0}^{\frac{\pi}{3}} 36x \cos x\,dx \le 2\pi^2$. **SCORE: _____ / 15 PTS**

FOR $0 \le x \le \frac{\pi}{3}$, $\underbrace{\frac{1}{2} \le \cos x \le 1}$ 3

$$\underbrace{18x \le 36x\cos x \le 36x}\ 3$$

$$\underbrace{\int_0^{\frac{\pi}{3}} 18x\,dx \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le \int_0^{\frac{\pi}{3}} 36x\,dx}\ 3$$

$$2\ \underbrace{9x^2\Big|_0^{\frac{\pi}{3}}} \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le \underbrace{18x^2\Big|_0^{\frac{\pi}{3}}}\ 2$$

$$1\ \underbrace{\pi^2 = 9\left(\tfrac{\pi^2}{9}\right)} \le \int_0^{\frac{\pi}{3}} 36x\cos x\,dx \le \underbrace{18\left(\tfrac{\pi^2}{9}\right) = 2\pi^2}\ 1$$